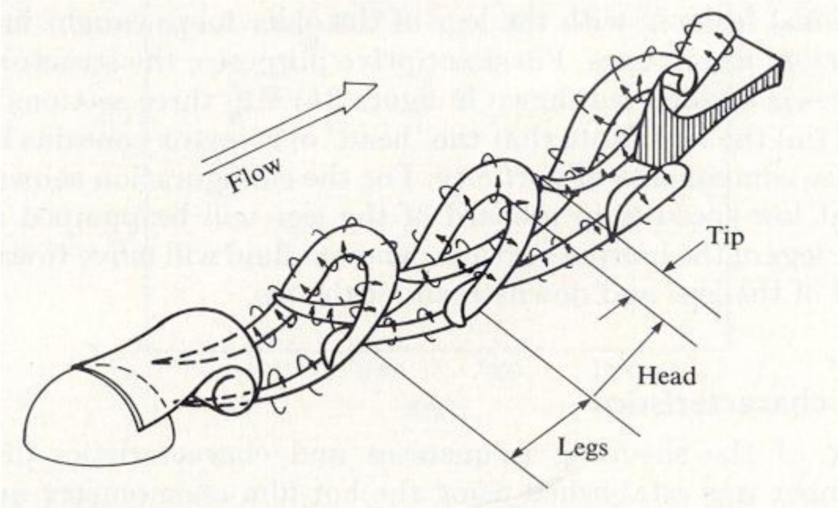

Acarlar & Smith ,1987

Tracking Stages of Transition in Couette Flow Analytically

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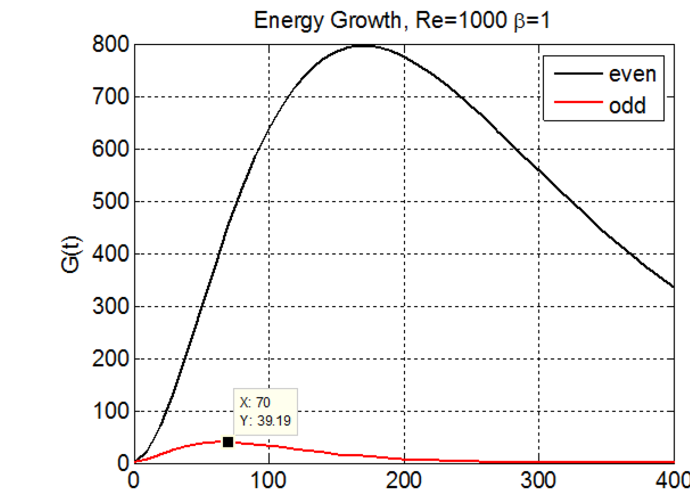

Figure 4. Energy growth Re=1000, comparison between even (2 vortices) and odd (4 vortices)

Critical Reynolds numbers for Transition

Theoretical values obtained by Linear Stability Theory (LST) disagree with the corresponding experimental values

Flow	Theoretical (LST)	Experimental
Pipe Poiseuille	∞ (Stable)	~2000
Plane Poiseuille	5772	~1000
Plane Couette	∞ (Stable)	~360

Table 1. Critical Reynolds numbers for transition, Theory vs. Experiment

Transient Growth (TG)

A possible explanation for the failure of the LST may lie within the Transient Growth mechanism according to which an infinitesimal disturbance may initially grow and only ultimately decay. During this growth, nonlinear effects may become considerable and instability may occur

Linear Optimal Disturbances Maximizing TG

The linear disturbance yielding the optimal energy growth corresponds to a Counter-Rotating Vortex Pair (CVP) – a pair of elongated streamwise vortices, generating streaks of high/low velocity. The optimization is performed on the gain (*G*) of the disturbance kinetic energy (*E*)

$$E(t)=\int_V\left(u^2+v^2+w^2\right)dV;\qquad G=\frac{E(t)}{E(0)}$$

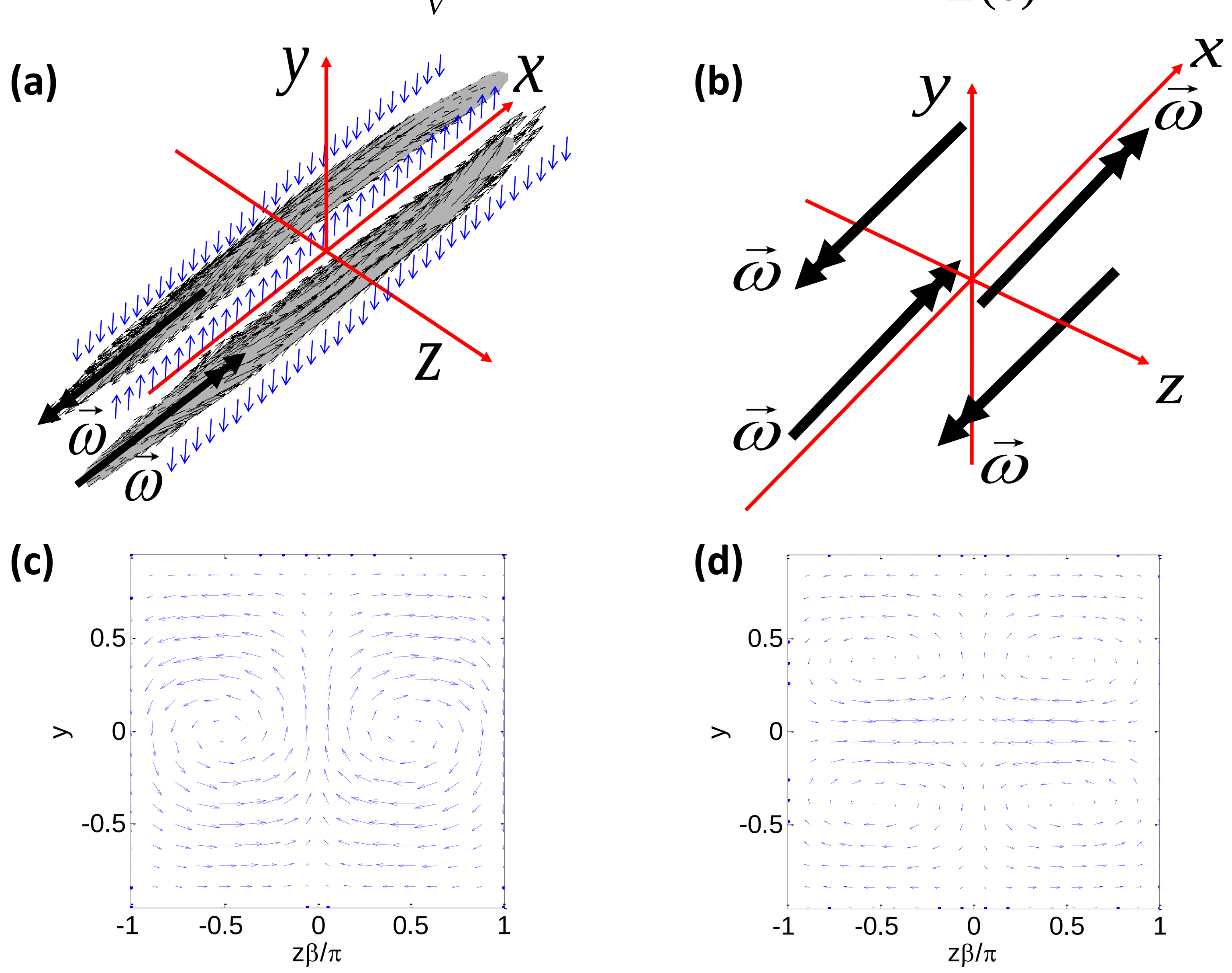

Figure 1. (a) single CVP = 2 vortices; (b) 2 CVPs = 4 vortices; (c,d) Cross-section velocity of single CVP (c) and 2 CVPs (d)

Research Aim

Propose an analytical approximation of the linear TG mechanism in Couette flow and utilize it to predict nonlinear transition to turbulence in Couette flow

Mathematical Method

(a) Analytical approximation of linear TG:

- Streamwise independence + spanwise wavenumber β:

$$\vec q(t,y,z)=\vec{\bar q}(y)\exp\left[i(\beta z-\omega t)\right];\quad \bar q=(u;v;w;p)$$

- Modes obtained analytically from Orr-Sommerfeld (OS) and Squire (Sq) equations, 2 families are obtained:

$$\bar v=0;\ \omega_{sq}=-i\left(s^2+\beta^2\right)/\mathrm{Re}\qquad \omega_{OS}=-i\left(p^2+\beta^2\right)/\mathrm{Re}$$

$$\bar\eta_{even}=\cos(s_{even}y);\ s_{even}=\frac{(2n-1)\pi}{2},\qquad \bar v_{even}=\frac{\cosh(\beta y)}{\cosh\beta}-\frac{\cos(p_{even}y)}{\cos p_{even}};\quad \beta\tanh\beta+p_{even}\tan p_{even}=0,$$

$$\bar\eta_{odd}=\sin(s_{odd}y);\ s_{odd}=n\pi\qquad \bar v_{odd}=\frac{\sinh(\beta y)}{\sinh\beta}-\frac{\sin(p_{odd}y)}{\sin p_{odd}};\quad \beta\coth\beta-p_{odd}\cot p_{odd}=0$$

- Derive analytical expression for the energy based on <u>4 modes</u>, their coefficients are determined to maximize the growth

(b) Calculation of nonlinear interactions between the 4 modes using an asymptotic expansion:

$$\vec u=y\hat e_x+\varepsilon\vec u_1(t,y,z)+\varepsilon^2\vec u_2(t,y,z)+\ldots$$

(c) Secondary stability analysis of the modified baseflow

- Modified baseflow = Couette + TG; (TG = 4 modes + nonlinear)
- Floquet theory for a spanwise periodic baseflow $U_0(t,y,z)$
- Linear two-dimensional stability analysis

$$\vec q_d(t,x,y,z)=\exp\left[i(\alpha x-\omega t)\right]\cdot\sum_{k=-\infty}^{\infty}\vec{\bar q}_k(y)\exp(i\beta kz);\quad \vec q=(u_d;v_d;w_d;p_d)$$

Verification

Very good agreement between 4 modes and the optimal. *Physical mechanism* of TG understood from the mutual initial cancellation of the modes

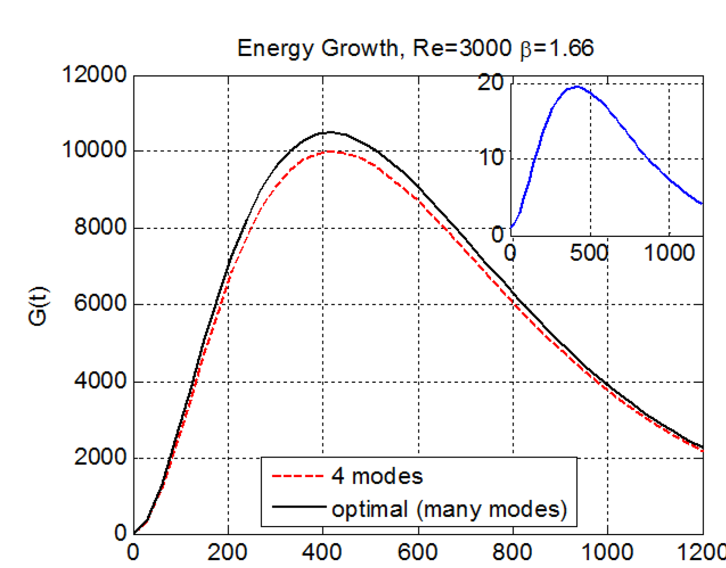

Figure 2. Energy growth for Re=3000 and β=1.66; optimal vs. 4 modes (analytical); In the inset: 2 modes (analytical)

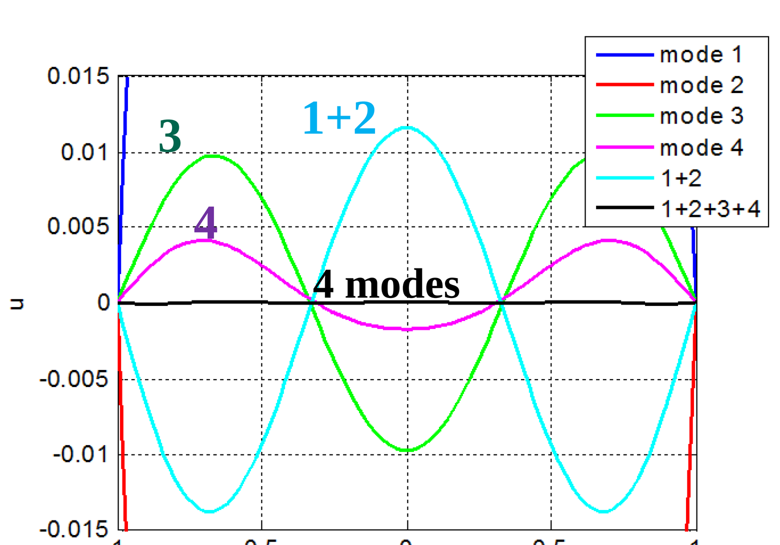

Figure 3. Initial streamwise velocity, 4 modes

Acknowledgments

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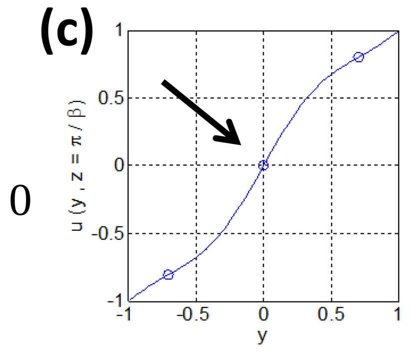
Results

Current study focuses on the odd disturbance to demonstrate that significant growth is not essential (Fig. 4)

Secondary instability verified by obtaining transition in DNS (Channelflow, Gibson(2012))

Physical mechanism of transition:

- The creation of a wall-normal inflection point at y=0 (Fig. 5c) $\frac{\partial^2 u}{\partial y^2}=0$



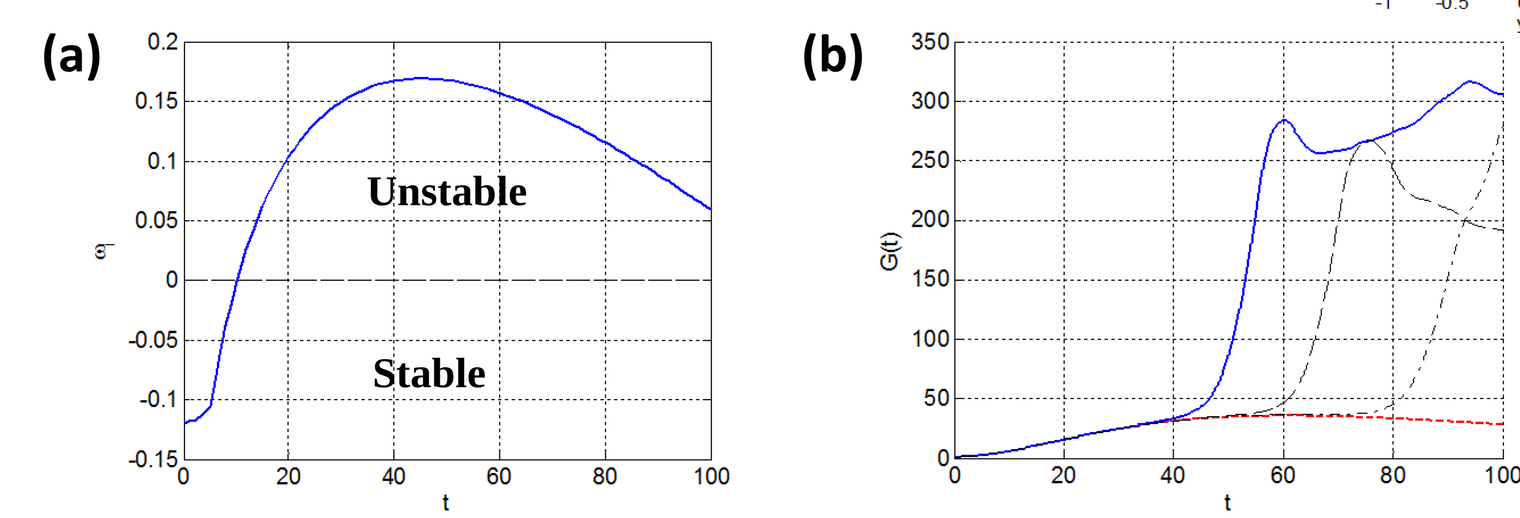


Figure 5. Odd disturbance Re=1000; (a) Secondary stability analysis; (b) Transition scenarios obtained by DNS; (c) Inflectional streamwise velocity profile at T=10

Following the process analytically

Compare analytical expressions to DNS:

$$\vec u=y\hat e_x+\varepsilon\vec u_1(t,y,z)+\varepsilon^2\vec u_2(t,y,z)+\delta\vec u_d(t,x,y,z)+\ldots$$

Couette + 4 modes + nonlinear + secondary

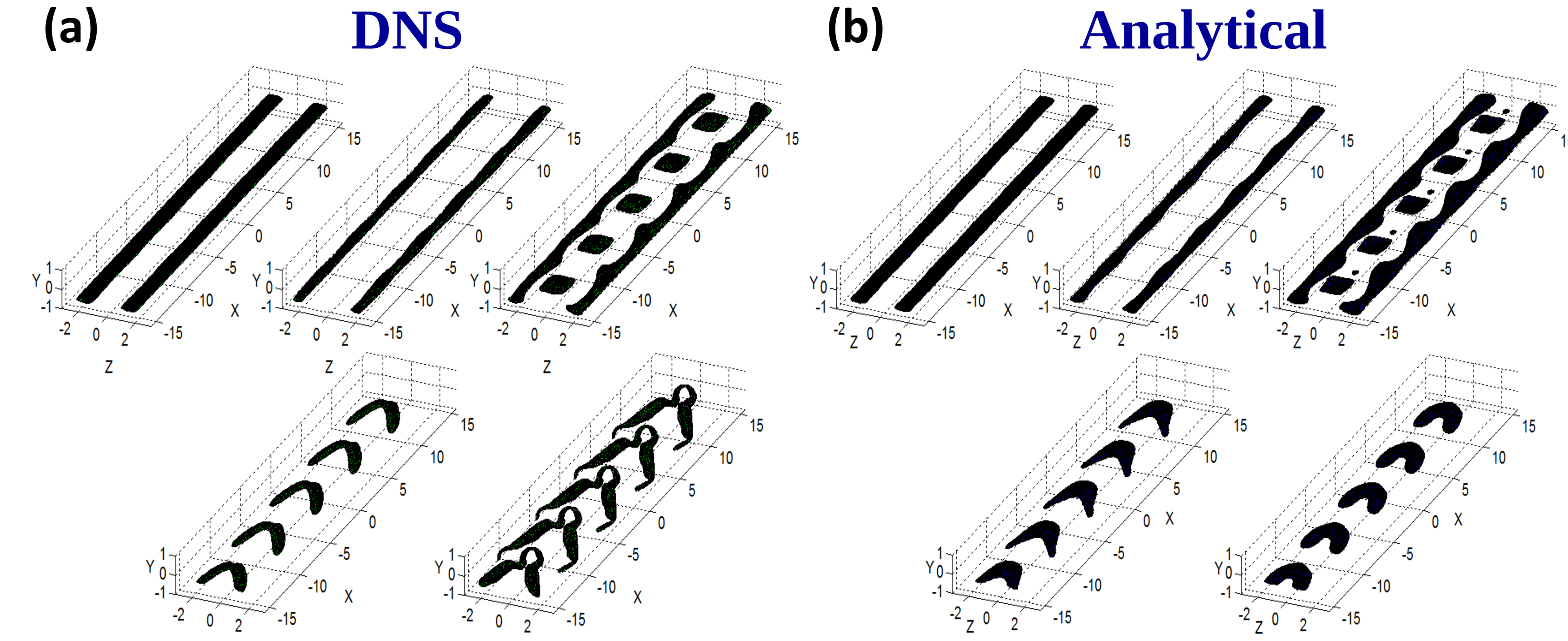


Figure 6: Vortical structures from DNS (a) vs. analytical (b)

Summary

- 4 modes are sufficient to model TG
- Most of the transition stages captured analytically
- Maximal growth is <u>not</u> essential for transition
- The role of the TG is to generate inflection points
- Transition dominated by a packet of hairpins

References

Karp, M. and Cohen, J. "Tracking Stages of Transition in Couette Flow Analytically", Accepted for publication in J. Fluid Mech.