Secondary instability of Transient Growth in Couette Flow

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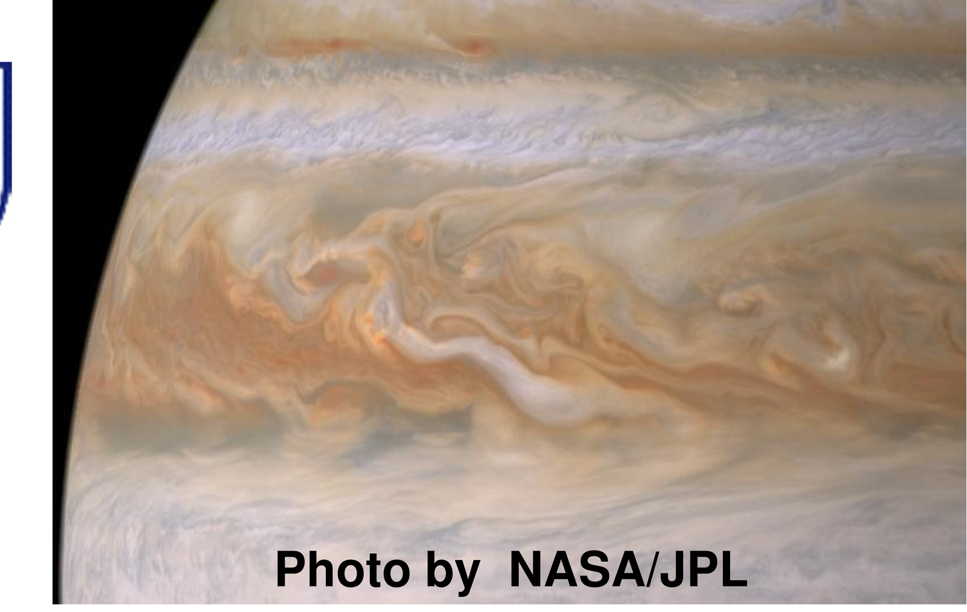
Research supported by the Israeli Science Foundation under Grant No. 1394/11



Motivation

- **Linear Stability Theory (LST) is unable to predict transition**

Flow	Theoretical (LST)	Experimental
Pipe Poiseuille	∞ (Stable)	~2000
Plane Poiseuille	5772	~1000
Plane Couette	∞ (Stable)	~360

Table 1. Critical Reynolds numbers for transition, Theory vs. Experiment

Transient Growth (TG)

- **A mechanism where infinitesimal disturbances grow in a stable flow. During this growth, the baseflow can be modified significantly and instability may occur.**
- **Most efficient TG occurs for streamwise independent vortices**

Research Aim

- **Study the secondary instability of TG in Couette flow and utilize it to predict nonlinear transition to turbulence**

Mathematical Method

- **Analytical approximation of linear TG using 4 modes**
- **Calculation of nonlinear interactions between the 4 modes**
- **Secondary stability analysis of the modified baseflow**

$$\boldsymbol{u} = y\hat{e}_x + \varepsilon \boldsymbol{u}_L(t,y,z) + \varepsilon^2 \boldsymbol{u}_{NL}(t,y,z) + \delta \boldsymbol{u}_d(t,x,y,z) + \ldots$$

Couette + *4 modes* + *nonlinear* + *secondary*

- **Long time correction of *up* using solvability condition**

$$\boldsymbol{u}_d = A_0 \tilde{\boldsymbol{u}}_d(t,y,z)\exp\left\{ i\left[\alpha x - \int_{t_0}^{t}\left(\omega(\tau) - \frac{iN}{M}\right)d\tau\right]\right\}$$

Amplitude, *Eigenfunction*, *Streamwise wavenumber*, *Eigenvalue*, *Long time correction*

Results

- **Optimal disturbances**
  - **Even TG – Sinuous, max. spanwise shear ($\beta \approx 4$)**
  - **Odd TG – Sinuous, max. spanwise shear ($\beta \approx 4$)**
  - **Odd TG – Varicose, max. spanwise shear ($\beta \approx 2$)**
- **Secondary instability verified by obtaining transition in 'Channelflow' DNS (Gibson, 2012)**

Summary

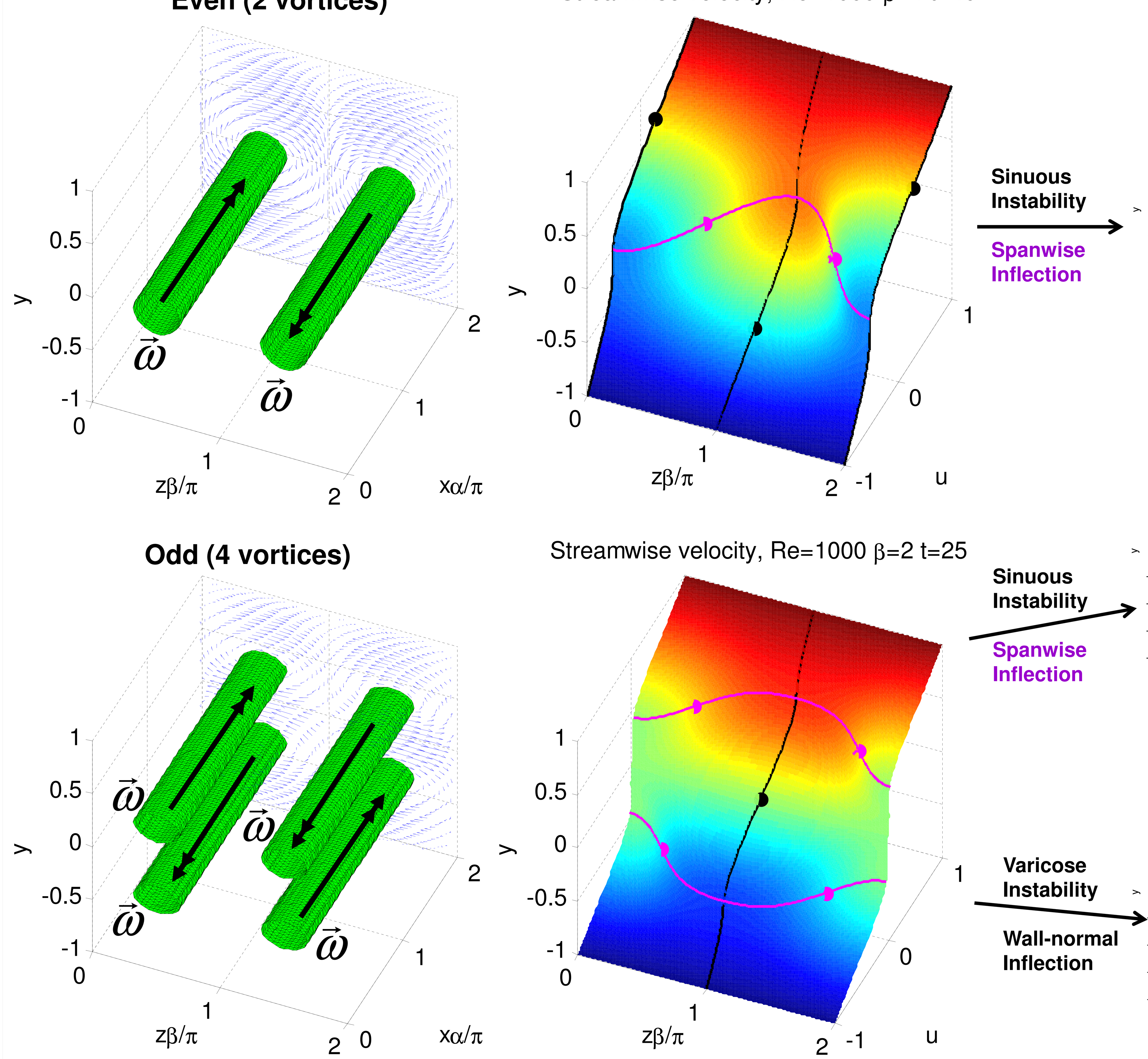
- **Maximal growth is not essential for transition**
- **The role of the TG is to generate inflection points**
- **Optimal disturbances occur at maximal shear**
- **Most transition stages are captured analytically**

References

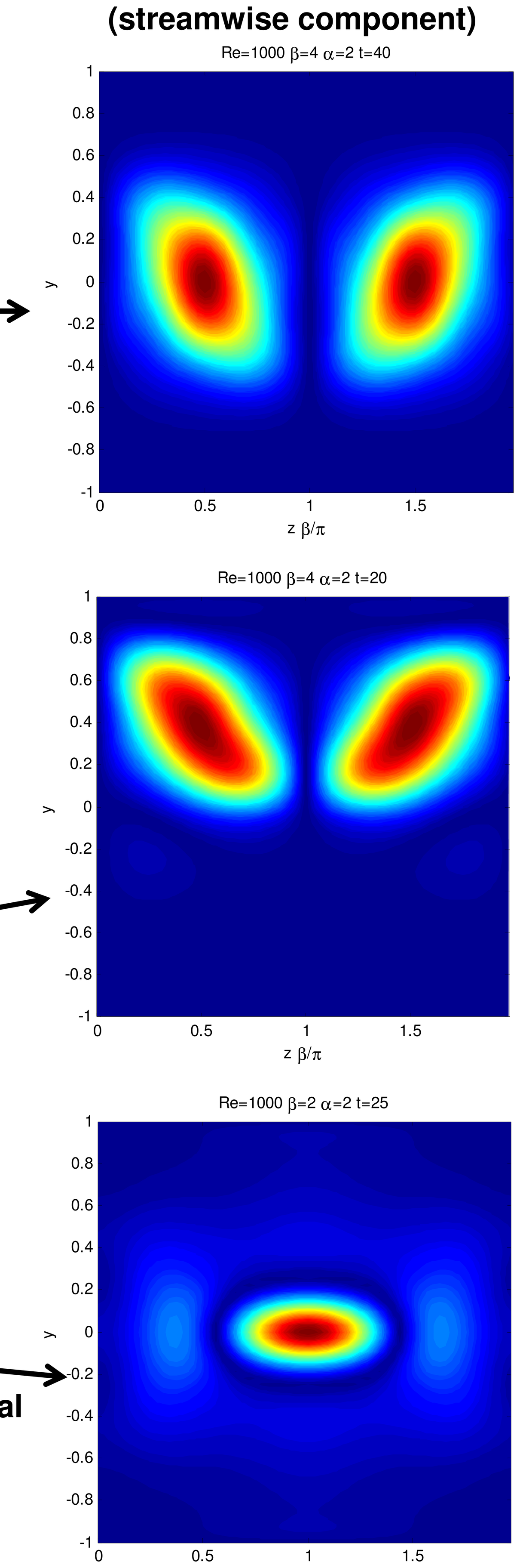
"Tracking stages of transition in Couette flow analytically" Karp, M., and Cohen, J., J. Fluid Mech., 748, 2014, pp 896 - 931.

Even (2 vortices) — Streamwise velocity, Re=1000 β=4 t=40

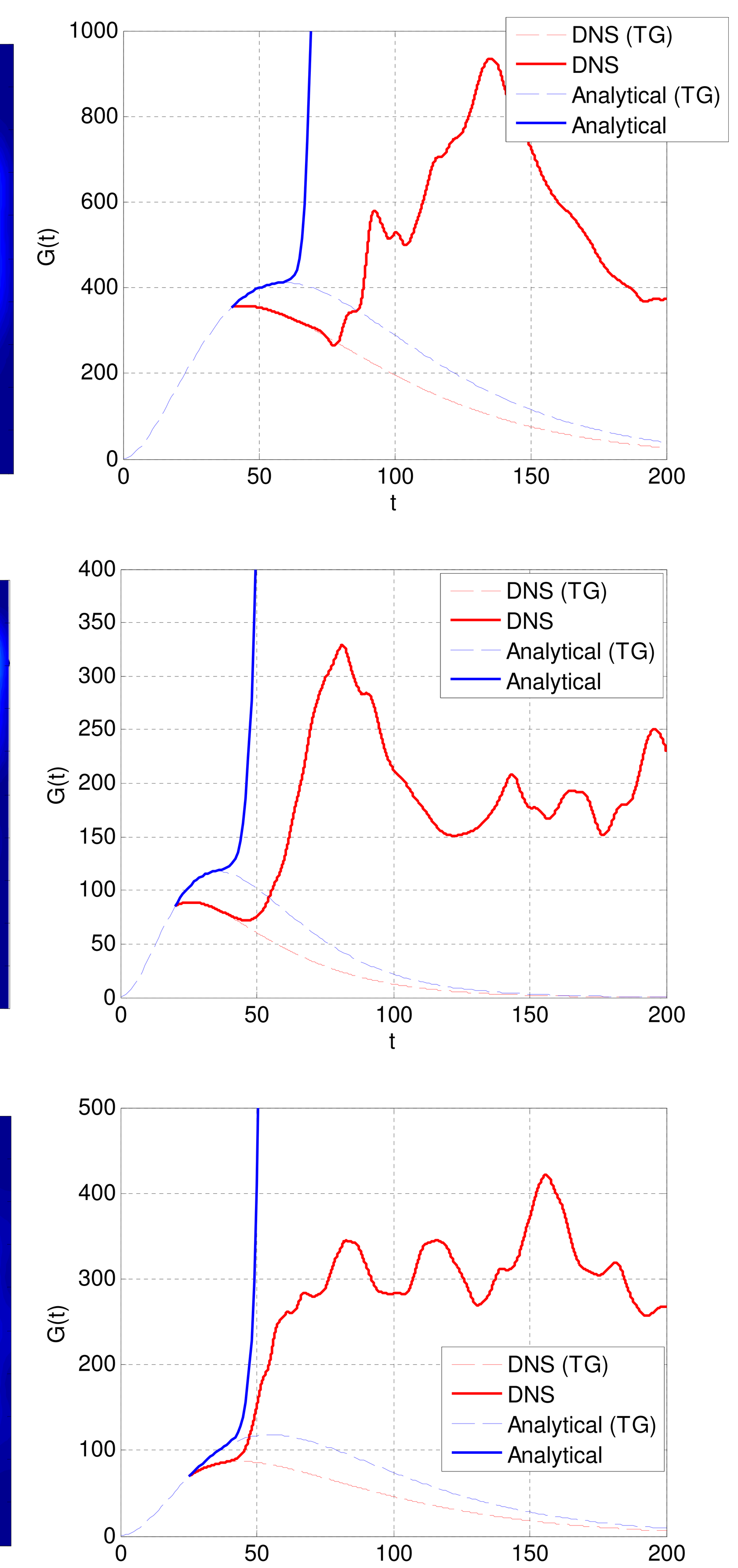
Odd (4 vortices) — Streamwise velocity, Re=1000 β=2 t=25



Secondary disturbance (streamwise component)



Energy during transition



Isosurfaces of the Q definition during transition