Stability of streaks in shear flows

Michael Karp and Jacob Cohen
Faculty of Aerospace Engineering
Technion - Israel Institute of Technology, Haifa 32000, Israel

Research supported by the Israeli Science Foundation under Grant No. 1394/11

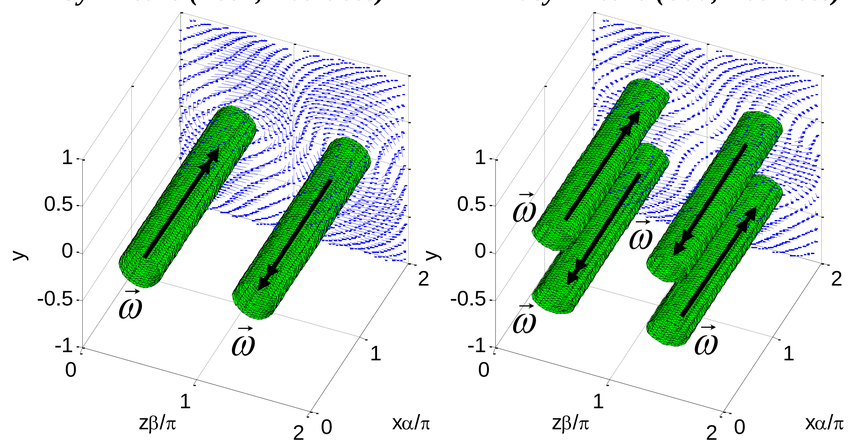
Research Aim

- *Study the secondary instability of TG in Couette flow and utilize it to predict nonlinear transition to turbulence*

Transient Growth (TG)

- *A mechanism where infinitesimal disturbances grow in a stable flow. During this growth, the baseflow can be modified significantly and instability may occur.*
- *Most efficient TG occurs for streamwise independent vortices*

Symmetric (Even, 2 vortices) *Antisymmetric (Odd, 4 vortices)*



Mathematical Method

- *Analytical approximation of linear TG using 4 modes*
- *Calculation of nonlinear interactions between the 4 modes*

$$\boldsymbol{U}_0(\tau,y,z)=y\hat{e}_x+\varepsilon\boldsymbol{u}_L(t,y,z)+\varepsilon^2\boldsymbol{u}_{NL}(t,y,z)+\ldots$$

Couette + 4 modes + nonlinear

- *Secondary stability analysis of the modified baseflow*

$$\boldsymbol{u}=\boldsymbol{U}_0(\tau,y,z)+\delta\boldsymbol{u}_d(t,\tau,x,y,z)+\mu\delta\boldsymbol{u}_{dd}(t,\tau,x,y,z)+\ldots$$

TG baseflow + Secondary disturbance + correction term

- *Long time correction of u_d using solvability condition*

$$\boldsymbol{u}_d=A_0\tilde{\boldsymbol{u}}_d(t,y,z)\exp\left\{i\left[\alpha x-\int_{t_0}^{t}\left(\omega(\tau)-\frac{i}{\mathrm{Re}}\frac{N}{M}\right)d\tau\right]\right\}$$

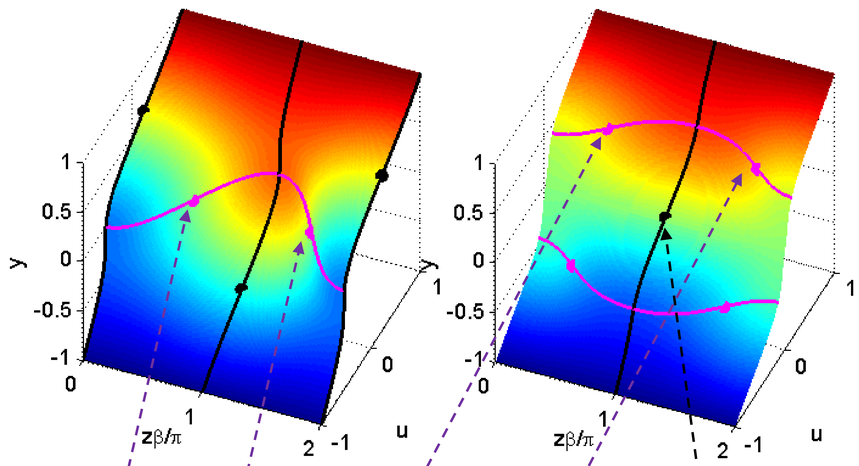
Amplitude
Eigenfunction *Streamwise wavenumber* *Eigenvalue* *Long time correction*

- *Secondary instability verified by obtaining transition in 'Channelflow' DNS (Gibson, 2012)*

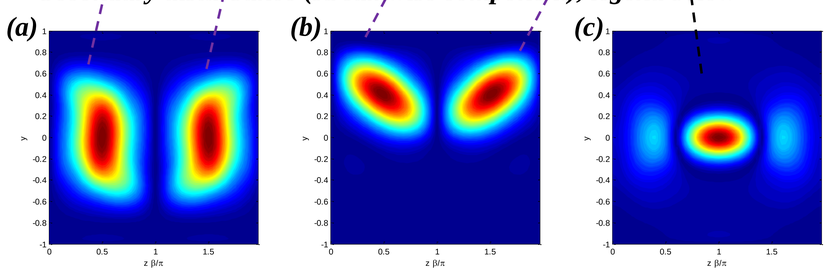
Results

- *Formation of streamwise velocity streaks containing inflection points*

Even TG (2 vortices) *Odd TG (4 vortices)*

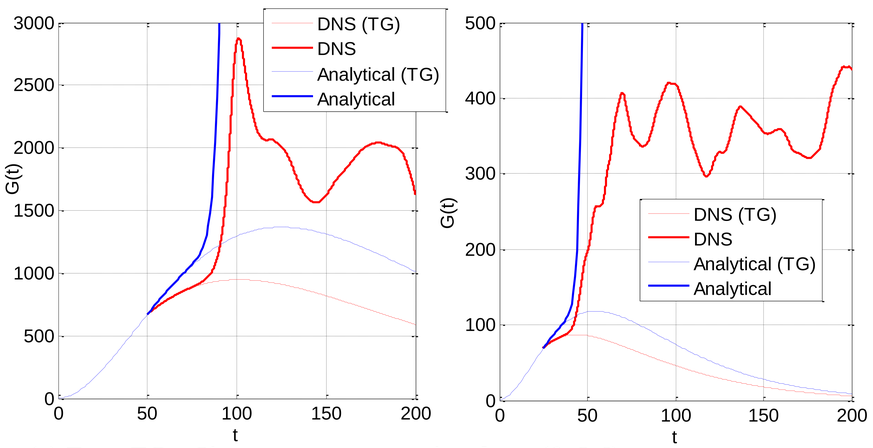


- *Secondary disturbance (streamwise component), legend below*



- *Energy growth during transition*

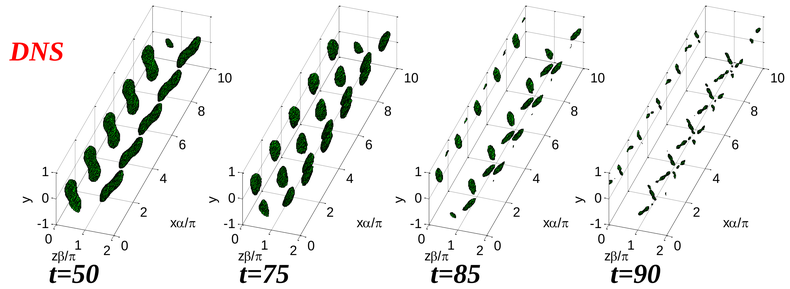
Even TG (2 vortices) - sinuous *Odd TG (4 vortices) - varicose*



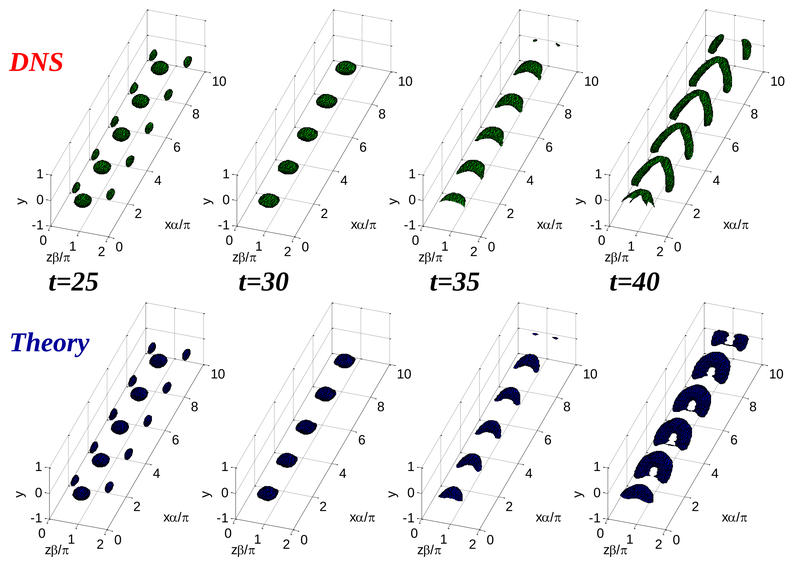
(a) Even TG – Sinuous, max. spanwise shear (β=3.6)
(b) Odd TG – Sinuous, max. spanwise shear (β=3.5)
(c) Odd TG – Varicose, max. wall-normal shear (β=1.6)

- *Vortical structures (Isosurfaces of the Q def.) during transition*

Even TG (2 vortices) - sinuous



Odd TG (4 vortices) - varicose



Summary

- *Maximal growth is <u>not</u> essential for transition*
- *The role of the TG is to generate inflection points*
- *Optimal disturbances occur at maximal shear*
- *Most transition stages are captured analytically*

References

"Tracking stages of transition in Couette flow analytically" Karp, M., and Cohen, J., J. Fluid Mech., 748, 2014, pp 896 – 931.